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Mathematical Analysis of a Fractional COVID- 19 Modal Applied to India

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Abstract:
The covid -19 had a huge impact on India. The rise in the affected population caused a shortage of hospital beds and ventilators and a lack of medical personnel, especially in the public health sector making India the third-worst affected country worldwide. Quarantine was done to maintain social distancing and control the rapid spread of the virus. But due to the increasing number of affected people in India, quarantine became more difficult. This led to a decrease in the country's stability. So, this paper intends to propose a fractional order model to check the stability in India.

Key World: Covid -19, Fractional order model, Population, stability, India.

Introduction:
Information about the spread of infectious diseases and its analysis due to the continuous spread had put a lot of loads on India. It had become the fundamental duty to see the ongoing diseases and the health of the people, but according to India, it is very difficult to separate people.
This SIR model, based on the compartment model was first introduced by Carmack and McKendrick in the epidemiology status and spacious. The faith organization banned the people of the country from moving across countries which affected nearly the whole country. Due to the nonavailability of proper medicine and the continuous spread, maintaining social distancing became the only way to control the spread of the disease for reducing the infection between people. Quarantine was found to be a good solution for this. For this, identification had to be done for every person admitted to the hospital and additional family members had to be informed about it such that the family can be tested for corona. But due to the increasing number of affected people in India, quarantine became more difficult. The symptoms of the disease spread and the problem was the same for all and Covid -19 remained a dangerous threat in India. The government of India allowed people to come back to their home place and then send them directly to quarantine. The India report also used the Delhi case in the model and discussed the disease with the people of India. By studying the Covid-19 Pandemic in India: A Mathematical Study by Sudhanshu Kumar Biswas Et Al[1] and considering the model proposed by Faïçal Ndaïrou and Delfim F.M. Torres in Mathematical Analysis of a Fractional Covid 19 Model Applied to Wuhan, Spain, and Portugal[2], this paper intends to propose a fractional-order model to check the stability of infected people in India.

Preliminaries on Fractional Calculus:

Fractional derivative definitions and then recall some basic Properties useful to study the fractional-order model. The Caputo fractional derivative of order and then recall some basic properties useful to study the fractional order model.

The Caputo fractional

derivative of order$\alpha \epsilon$ $(0\ 1)$of a function x: $[0+\infty] \rightarrow R$ is given by

$$c_{D^{\alpha}}\, x(t) \;= \frac{1}{T(1-\alpha)} \int_0^t \quad (t-s)^{-\alpha} \;\; x^{t\,(s\,)ds}$$

Where T (1-α) =$\int_0^t \quad t^{-\alpha}$ exp(-t) Euler Gamma function. The value of the Caputo fractional derivative of the function x at point t involve all the value of x^t(s) for $S \in\ [0,t]$.

fractional order initial value problem:

$\{c_{D^{\alpha}}\ X(t)\} = f(X),$

X (0) = Xo, Xo ∊ R^n.

Lemma1. Assume that the vector function f satisfies the following condition

1. F(X) and $\frac{\partial f\,(X)}{\partial X}$ are continuous for all X ∈ R^n.
2. F(X) ≤ ω + λ X for all X∊ R^n where ω or λ are two positive constants.

Lemma2. Generalized mean value theorem – suppose that the function x(t) and $c_{D^{\alpha}}$ x(t) are both continuous on $[0,b]$. Then

X(t) = X(0) + $\frac{1}{T(\alpha)} x(n) t^{\alpha}$, 0< $n < t, \forall t \in\ [0\ b]$

We have that if $c_{D^{\alpha}}$ x(t) > 0 for all t ∈ $[0\ b]$ then the function x is strictly decreasing.

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The Considered Fractional- order COVID-19 Model:

The total population N is constant, along the period under study and made up with eight sub –population of dynamic transition as different stage of transmission of the virus to individual grouped into compartmental class

S(t) + E(t) + I(t) + P(t) +A(t) +H(t) + R(t) + F(t) = N

Where, S(t) =The susceptible individual at time t.

E(t) = the exposed individuals.

I(t) =the symptomatic and infectious individuals.

P(t) = the super- spreaders individuals.

A(t) = the infection but asymptomatic individuals.

H(t) = the hospitalized individuals.

R(t) = the recovery individual. And

F(t) = the dead individuals or fatality class.

The Caputo fractional- order system that describe the dynamics transmission is given by-

$c_{D^{\alpha}}$ S(t) = -$\beta\frac{I(t)}{N}$ S(t) - I$\beta\frac{H(t)}{N}$S(t) -$\beta'\frac{P(t)}{N}$ $S(t)$,

$c_{D^{\alpha}}$ E(t) =$\beta\frac{I(t)}{N}$S(t) +I$\beta\frac{H(t)}{N}$S(t)=$\beta'\frac{P(t)}{N}$S(t) –kE(t),

$c_{D^{\alpha}}$ I(t) =kp_1E(t) – ($\gamma_{\circ}$ +γ_1)I(t)- δ_iI(t),

$c_{D^{\alpha}}$p(t) =kp_2E(t)-($\gamma_{\circ}$+γ_i) $p(t) - \delta_p\, p(t)$,

$c_{D^{\alpha}}$ A(t) =k(1-ρ_1- ρ_2) $E(t)$,

$c_{D^{\alpha}}$H(t) = γ_aI(t) + P(t) - γ_r $H(t)$ - δ_hH(t),

$c_{D^{\alpha}}$ R(t) = γ_iI(t) + P(t) + γ_r H(t),

$c_{D^{\alpha}}F(t)$ = δ_iI(t) + $\delta_p P(t) + \delta_h H(t)$,

The expression$\beta\ \frac{I}{N}S + I\beta\frac{H}{N}S +\ \beta'\frac{P}{N}S$ *represent the* force of infection of the virus that is the transmission term or the effective contact between susceptible individual (S) and infectious symptomatic individual(l), super-spreaders individual(P) and hospitalized ones(H) here βquantifies the human-to-human transmission coefficient per unit of time (days) per person, β'quantifies a high transmission coefficient due super-spreaders, N quantifies the relative transmissibility of hospitalized patients.

. k is the rate at which an individual leaves the exposed class by becoming infectious (symptomatic, super-spreader or asymptomatic,

. ρ_1 *is the* proportion of progression from exposed class E to symptomatic infectious class I

. ρ_2 is relative very low rate at which exposed individuals become super- spread.

. 1- $\rho_1 - \rho_2$ *is the* progression from exposed to asymptomatic class.

.γ_a *is the average* rate at which symptomatic and super- spreaders individual become hospitalized.

. γ_i *i*s the recovery rate without being hospitalized.

. γ_r is the recovery rate of hospitalized patients.

. δ_i denote the disease induced death rates due to infected individuals.

. δ_p denote the disease induced death rates due to super- spreaders individuals.

. δ_h denote the disease induced death rates due to hospitalized individual.

Existence and Uniqueness of positive Solution:

It is denoted by- $R_+^8 = \{X \in R^8 : X \geq 0\}$

And let X(t) = $\{S(t), E(t), P(t), I(t), A(t), H(t), R(t), F(t)\}^T$ then the system as follows-

c_{D^α} X(t) = F(X(t)),

Where F(X) =$\Big(-\beta\frac{I}{N}S - I\beta\frac{H}{N}S - \beta'\frac{P}{N}S\ \beta\frac{I}{N}S + I\beta\frac{H}{N}S + \beta'\ \frac{P}{N}S -$
$kE\ k\rho 1\ E{-}\gamma a{+}\gamma iI{-}\ \delta i\ I\ k\rho 2E{-}\gamma a{+}\gamma iP{-}\ \delta p\ P\ k1{-}\rho 1{-}\rho 2E$ ⍰ $aI{+}P{-}\gamma r\ H{-}\ \delta hH$
⍰ $iI{+}P{+}\gamma r\ H$ ⍰ $iI{+}\delta p\ P{+}\ \delta h\ H$

We consider non- negative initial conditions

S (0) ≥ 0 , $E(0) \geq 0, I(0) \geq 0, P(0) \geq 0, A(0) \geq 0, H \geq 0, R\ \geq 0, F \geq 0$,in condition,

A_1
=(0 0 $-\beta$ 0 0 β 0 0 0 $-\beta'$ 0 $-$
1□ □′0 1□ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 00 0 00 0 00 00
00 00 000000 0 00 0 00 00 00

A_2
= (0 0 0 0 0 0 0 0 0 $-k$ 0 0 0 0 0 0 0 $k\rho_1$ $-\omega_i$ 0 0 0 0 0 0

where $\omega_e\ = k(1-\rho_1-\rho_2)$; $\omega_i\ = \gamma_a\ + \gamma_i\ + \delta_{p;}$ *and* $\omega_h + \gamma_r\ +$
δ_i *thus*

we can rewrite the vector form function F as-

F(X) = $\frac{S}{N}A_1X + A_2X$.

Stability Analysis: HOW to proof of Global stability-

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Disease free equilibrium point (DFE) = (N,0,0,0,0,0,0,0)

Using the next generation matrix approach is given by-

$$R_0 = \frac{\beta\rho_1(\gamma_a l + \omega_h)}{\omega_i\,\omega_h} + \frac{\beta\gamma_a l + \beta\gamma_a l + \beta'\omega_h)\rho_2}{\omega_\rho\,\omega_h}.$$

Numerical Simulations:

The study the dynamical behaviour of infected individuals(I), Super-spreaders (P, hospital sized individual(H), and the cumulative cases of infections (I+P+H), Which describe the COVID-19 dynamics transmission of INDIA.

Population Size Initial Condition and parameters:

The total population size under study reflects specificity on the spread of COVID-19 on each territories considered. Therefore we consider N = 1352642280
S_0 =1352642280 E_0=1131 I_0=482 P_0=647 $A_0 =$ 506 $H_0 = 657$
R_0=2.414190966 F_0=0
for INDIA, the following value of parameters are –

Table- *The model parameter and their sensitivity indices for covid-19 Pandemic in India estimated from the data from March to April 2020.* (Source:[1])

Parameters	Value	Sources	Sensitivity India

π	67446.82054day-1	(14)	_
β	0.88689day-1	Estimated	1.0000
μ	0.0000391day-1	(14)	-0.00065
σ_a	0.24176day-1	Estimated	0.44295
σ_i	0.24757day-1	Estimated	-0.13547
σ_q	0.26556day-1	Estimated	-0.30743
	0.05090day-1	Estimated	-0.02561
γ_i	0.05071day-1	Estimated	-0.00435
γ_q	0.07048day-1	Estimated	-0.03394
γ_h	0.26267day-1	Estimated	-0.10165
η_i	0.39787day-1	Estimated	0.00255
η_q	0.06891day-1	Estimated	-0.07377
δ	0.67047	Estimated	0.76318
ρ	0.94	Assumed	-0.12070
q1	0.90	Assumed	-0.60436
q2	0.48576	Estimated	-0.94461
d	4.488983	Estimated	-22.1669122
R_0			

Used the data estimated parameter made from initial conditions. Sensitivity of the parameter on R_0 of positive (R_0 increases) or negative (R_0 decreases) sign is determined by the magnitude of index, i.e., higher the magnitude, higher the sensitivity.

Conclusion:

After some loss of stability in the country due to the effect of the coronavirus, it has become the fundamental duty of the country to see the ongoing diseases and health of the people. By using the data of people infected in India, this paper proposes a fractional-order model to check the stability of infected people in India. The model will also be helpful in proving the Global stability of India and ensuring its balance in the future as well.

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